Gage Linearity and Accuracy

Summary

The *Gage Linearity and Accuracy* procedure is designed to estimate the accuracy of a measurement system. In contrast to procedures such as the *Average and Range Method* and the *Anova Method*, which estimate the variability or *precision* of a measurement system, this procedure is concerned with how *accurate* a system is on average.

To quantify the above concepts, suppose that a measurement process when repeated on the same part many times yields measurements that come from a population with mean μ and standard deviation σ. The bias of the measurement process is defined as the difference between the mean of that population and the true value being measured, i.e.,

$$\text{bias} = \mu - \text{true value} \qquad (1)$$

A process is said to be "*accurate*" if the bias is small. In contrast, precision is directly related to σ, with smaller values of σ being characteristic of "*precise*" processes. A poor measurement process could be accurate but not precise, or precise but not accurate.

On the other hand, if a measurement is known to have substantial bias, but that bias is consistent across all items being measured, then it may be possible to compensate for that bias by adjusting the measured values. *Linearity* is a term that refers to how much the bias changes throughout the normal operating range of a gage or other measurement instrument. If the bias is consistent, then the linearity will be small.

Sample StatFolio: *gagelinearity.sgp*

Sample Data:

The file *linearity.sgd* contains data from a typical gage linearity study, taken from the third edition of the Automotive Industry Action Group's (AIAG) reference manual on Measurement Systems Analysis, MSA (2002). A portion of the data in that file is shown below:

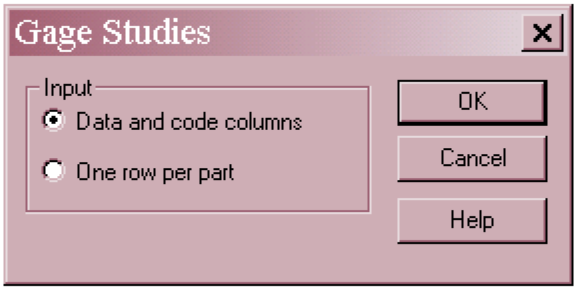
Part	*Reference*	*Measurement*
1	2	2.7
2	4	5.1
3	6	5.8
4	8	7.6
5	10	9.1
1	2	2.5
2	4	3.9
3	6	5.7
4	8	7.7
5	10	9.3

The file contains a total of n = 60 rows, consisting of m = 12 measurements by a single appraiser on each of g = 5 parts. The 5 parts have known reference values ranging from 2.00 to 10.00. The intent of the study is to determine how well the measurement system matches those reference values on average.

Note: Data reprinted from the Measurement Systems Analysis (MSA) Manual with permission of DaimlerChrysler, Ford and GM Supplier Quality Requirements Task Force.

Data Input

The first dialog box displayed by this procedure is used to indicate the structure of the data to be analyzed.



Input: The datasheet may be organized into either of two formats:

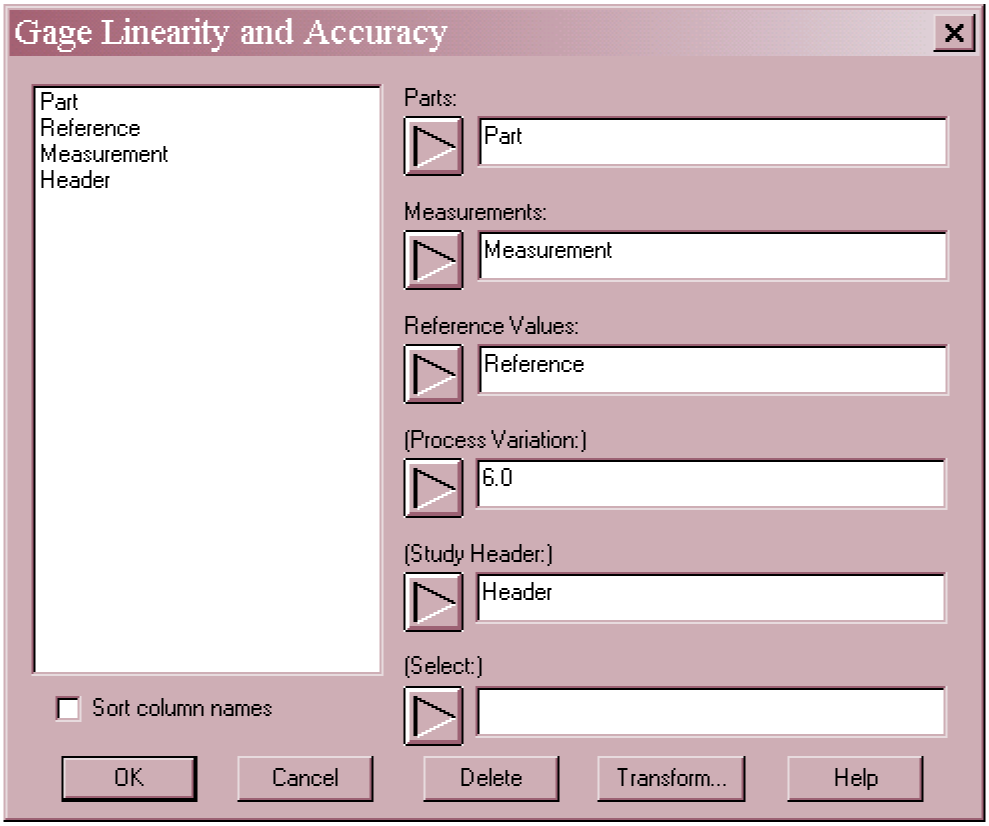
- *Data and Code Columns*: indicates that the datasheet contains a single column holding all the measurements. This is the type of data structure illustrated above.
- *One Row for Each Part*: indicates that the datasheet contains a single row for all measurements on a specific part. An example of this data structure is shown below:

Part	Reference	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5	Trial 6
1	2	2.7	2.5	2.4	2.5	2.7	2.4
2	4	5.1	3.9	4.2	5	3.8	3.8
3	6	5.8	5.7	5.9	5.9	6	6.1
4	8	7.6	7.7	7.8	7.7	7.8	7.7
5	10	9.1	9.3	9.5	9.3	9.4	9.4

The second dialog box displayed depends on the setting in the first dialog box.

<u>Data and Code Columns</u>

If you select *Data and Code Columns* on the first dialog box, the second dialog box requests the name of the column containing the measurements, as well as columns containing the part numbers, reference values, and an optional value for the process variation.



- **Parts**: numeric or non-numeric column indicating the an identifier for the item corresponding to the measurement in each row.

- **Measurements**: numeric column containing the measurements.

- **Reference Values**: the true values of the variable being measured for each of the parts.

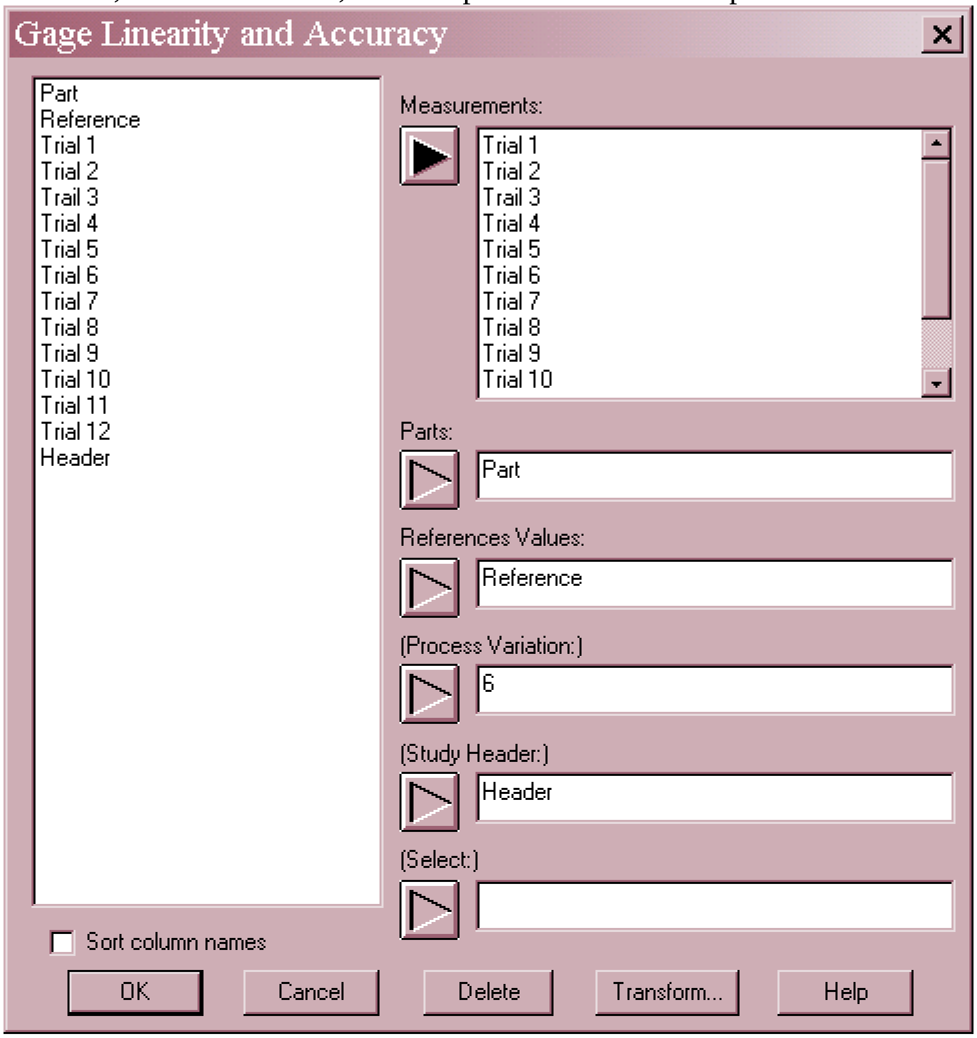
- **Process Variation:** the range over which the variable varies due to normal process variation. If not specified, the process variation is set to the range of the *Reference Values*.

- **Study Header**: optional header to be printed at the top of each output table.

- **Select**: subset selection.

One Row for Each Part

If you select *One Row for Each Part* on the first dialog box, the second dialog box requests the names of multiple columns containing the measurements, as well as columns containing the part numbers, reference values, and an optional value for the process variation.



- **Parts**: numeric or non-numeric column indicating an identifier for the item corresponding to the measurements in each row.

- **Measurements**: two or more numeric columns containing the measurements.

- **Reference Values**: the true values of the variable being measured for each of the parts.

- **Process Variation:** the range over which the variable varies due to normal process variation. If not specified, the process variation is set to the range of the *Reference Values*.

- **Study Header**: optional header to be printed at the top of each output table.

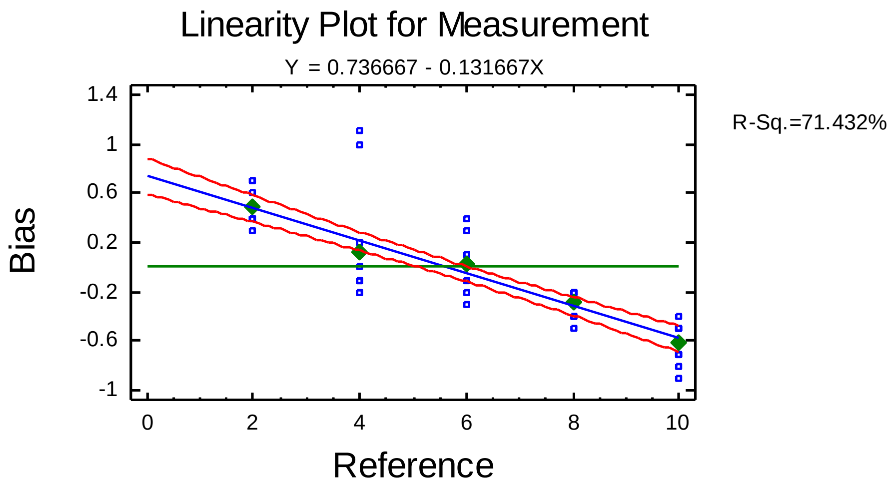
- **Select**: subset selection.

Linearity Plot

The initial step in analyzing data from this type of study is to calculate the differences between the observed measurements and the reference values:

$$y_{ij} = measurement_{ij} - reference\ value_{ij} \tag{2}$$

where $measurement_{ij}$ represents the i-th measurement made on the j-th part. The *Linearity Plot* shows these differences plotted against the references values x_{ij}:



Included on the plot are:

- **Blue squares**: y_{ij}, the differences between the original measurements and the corresponding reference values.
- **Green diamonds**: the average bias $\bar{y}_j$ at each reference value.
- **Blue line**: the least squares regression of the differences y_{ij} against the reference values x_{ij}. The equation of this line is shown along the top of the plot.
- **Red lines**: confidence limits for the regression line, at the level of confidence specified on the *Analysis Options* dialog box.
- **Green line:** horizontal line at bias equal to 0.
- **R.Sq.**: the R-squared statistic for the fitted model. This is a measure of how well the line fits the observed data.

In the current example, the bias changes quite a lot as the reference value changes, being positive at lower reference values and negative at higher reference values. If the measurement system has no bias, the green horizontal line will usually lie within the red confidence bands.

Analysis Summary

The *Analysis Summary* displays summary statistics for the estimated linearity and accuracy of the measurement system.

Gage Linearity and Accuracy - Part

AIAG Example
Parts: Part
Measurements: Measurement
Reference values: Reference
Process variation: 6

Number of Parts: 60
Total number of measurements: 60
Number of measurements excluded: 0
Range of reference values: 2 - 10

Estimates

	Estimate	*Percentage*
Bias	-0.0533333	0.889
Linearity	0.79	13.167

Bias model: 0.736667 - 0.131667x

Parameter	*Estimate*	*Stnd. Error*	*t Statistic*	*P-Value*
Intercept	0.736667	0.0725243	10.1575	0.0000
Slope	-0.131667	0.0109334	-12.0426	0.0000

Important information includes:

- **Bias**: the estimated average difference between the measured values and the reference values. The percentage bias is also calculated by:

  $$\%\ bias = |bias| \ /\ (process\ variation) \tag{3}$$

  and shows the magnitude of the bias relative to the normal operating range of the process.

- **Linearity**: the estimated change in the bias over the normal variation of the process. This is calculated by:

  $$linearity = |slope|\ (process\ variation) \tag{4}$$

  The percentage linearity is calculated by:

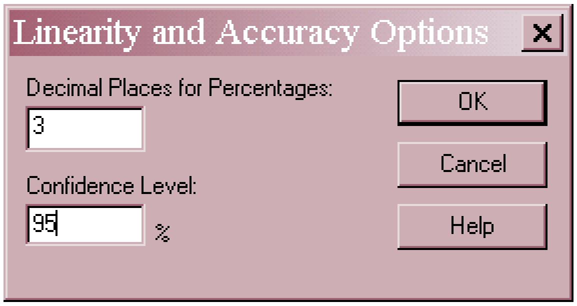
  $$\%\ linearity = linearity \ /\ (process\ variation) \tag{5}$$

  and shows how much the bias changes as a percentage of the process variation.

- **Bias model**: the equation of the fitted regression line and t-tests for the significance of the coefficients. Of particular interest is the P-value for the slope. If this value is small, (less than 0.05 if operating at the 5% significance level), then there is significant change in the bias over the range of the reference values.

In the current example, the average bias is small (less than 1%), but it changes by more than 13% over the operating range of the process. Notice also that the P-Value for the slope is well below 0.05. The process would therefore normally be classified as having a problem with linearity.

Analysis Options

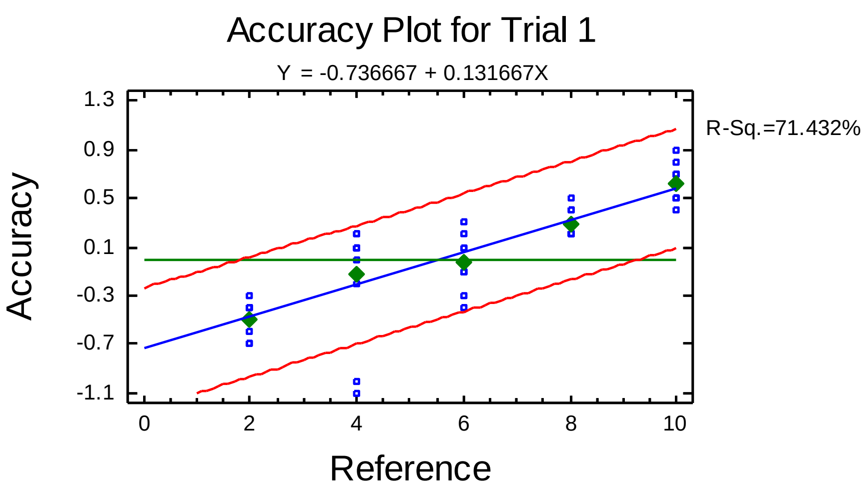


- **Decimal Places for Percentages**: number of decimal places to use when displaying percentage bias and linearity.

- **Confidence Level**: level used when plotting confidence bands on linearity and accuracy plots.

Accuracy Plot

The *Accuracy Plot* shows an estimate of the accuracy of the measurement process.



Accuracy is defined as the difference between the true value and the measured value:

$$accuracy = reference\ value - measured\ value \tag{6}$$

It is similar to the *Linearity Plot* with 2 major differences:

1. The Y-axis is inverted, since accuracy and bias are opposite in sign.
2. The bounds on the plot show the estimated prediction limits for individual observations, not confidence limits for the line.

You can sometimes detect the presence of outliers by examining points outside the prediction limits, such as the two very low points at a reference value of 4. If the data was entered in the *Data and Code Column* format, then the *Exclude* button on the analysis toolbar can be used to interactively remove any outliers and recalculate the regression line.

Lack-of-Fit Test

This pane shows the result of a test performed to determine whether a linear model is adequate to describe the change in bias with reference value.

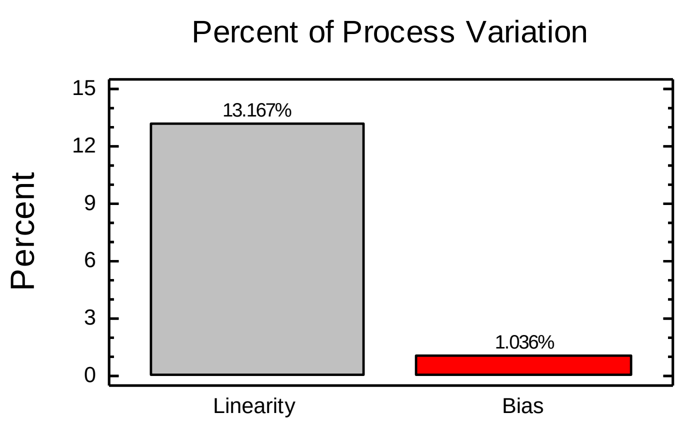
Analysis of Variance with Lack-of-Fit

Source	*Sum of Squares*	*Df*	*Mean Square*	*F-Ratio*	*P-Value*
Model	8.32133	1	8.32133	145.02	0.0000
Residual	3.328	58	0.0573793		
Lack-of-Fit	0.188	3	0.0626667	1.10	0.3579
Pure Error	3.14	55	0.0570909		
Total (Corr.)	11.6493	59			

Of primary interest is the *P-Value* on the *Lack-of-Fit* line of the table. If the P-value is small (less than 0.05 at the 5% significance level), then there is statistically significant “lack-of-fit”, meaning that a linear model is not adequate to describe the observed relationship. Such a result would imply the need for a curvilinear model. There is not significant lack-of-fit in the current example, since the P-Value is 0.3579.

Variation Barchart

This pane displays a barchart showing the percentage linearity and the percentage bias.



Calculations

- **Number of Measurements at j-th reference value**

$$n_{ij} \tag{7}$$

- **Average bias at j-th reference value**

$$\bar{y}_j = \frac{\sum_{i=1}^{n_j} y_{ij}}{n_j} \tag{8}$$

- **Bias**

$$\bar{y} = \frac{n_j \sum_{j=1}^{g} \bar{y}_j}{\sum_{j=1}^{g} n_j} \tag{9}$$

- **Regression**

The bias model is calculated by regressing the individual differences y_{ij} on the reference values x_{ij}. This has changed since Version 5 of STATGRAPHICS, where a weighted regression was performed on the average biases at each reference value. The estimated model will be different only if the study is unbalanced, i.e., if there are different numbers of measurements for different parts. The *R-Squared* statistic, however, will typically be smaller than in previous versions for all studies. This change was made to reflect changes in the latest edition of the AIAG manual.

The details of the regression calculations, including the t-tests, confidence bands, and lack-of-fit test, are given in the documentation for the *Simple Regression* procedure.