Date: April 13, 2012

From: Neil Polhemus, StatPoint Technologies, Inc.

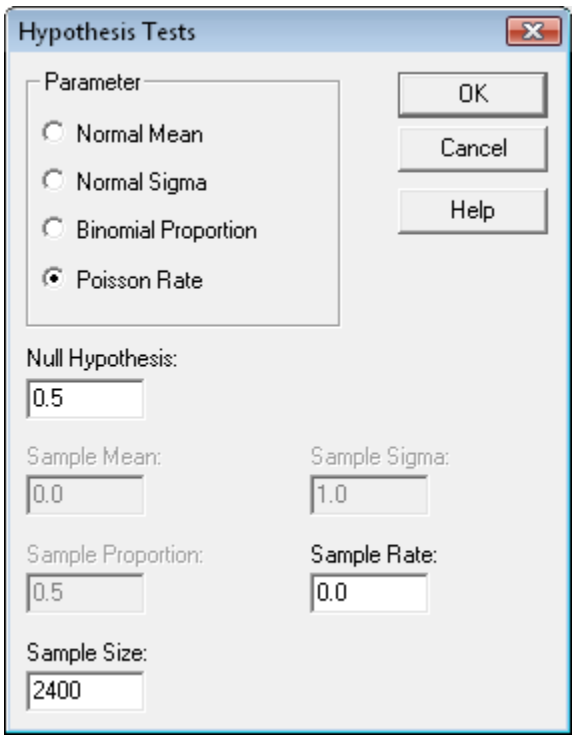
Subject: Data analysis with no failures

In a recent webinar, someone posed the following question:

Suppose I test 10 items for 240 hours and observe no failures. What can I say about the distribution of failure times?

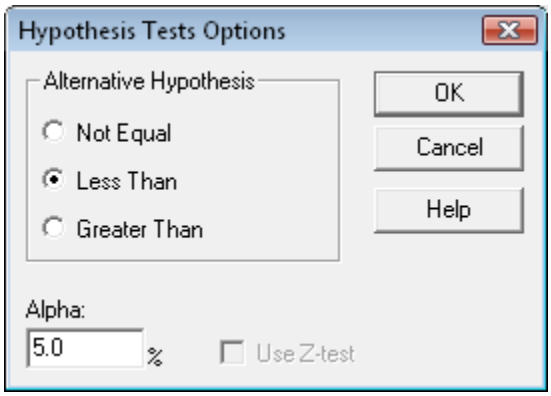
That's a very interesting question, for which we can say something if we make a strong enough assumption. If we assume that failures occur independently according to a Poisson process (which may be a reasonable assumption for high-reliability components during their normal operating period), then the number of failures that will occur if we observe the items for a total *time on test* of T hours follows a Poisson distribution with mean λT where λ equals the failure rate in failures per unit time. In addition, failure times during that period would follow an exponential distribution with mean $1/\lambda$. We can use Statgraphics to calculate an upper confidence bound on λ, and thus a lower confidence bound on the mean time to failure.

Step 1: Select *Describe – Numeric Data – Hypothesis* Tests from the main menu. Complete the first data input dialog box as shown below:



Select *Poisson Rate*, enter 0 for the *Sample Rate* (no failures have occurred) and 2400 for the *Sample Size* (which is the total sampling period over which no failures have been observed, T = 240*10). The *Null Hypothesis* field doesn't matter, since we won't test any hypothesis.

Step 2: Complete the second dialog box as shown below:



By selecting *Less Than* for the alternative hypothesis and 5% for *alpha*, we are requesting an upper 95% confidence bound for λ.

Step 3: The *Analysis Summary* appears below:

Hypothesis Tests
Sample rate = 0.0
Sample size = 2400

Approximate 95.0% upper confidence bound for rate: [0.00124822]

Based on what we have observed, we can be 95% certain that λ is no greater than 0.00125. Of course, λ is probably considerably smaller than that, but that's the upper bound.

A failure rate of 0.00125 failures per hour corresponds to a mean failure time of 1/0.00125 = 800 hours (this would be a 95% *lower* bound for the mean). Also, we can use the assumption of exponential failure times to get lower bounds for failure time percentiles.

Meeker and Escobar comment on this procedure in Section 7.7 of their book titled Statistical Methods for Reliability Data. As they point out, it's very difficult to get much useful information from a short test with zero or few failures. Also, without observing items for a long time, we can't be sure when the assumption of a constant failure rate will start to fall apart.